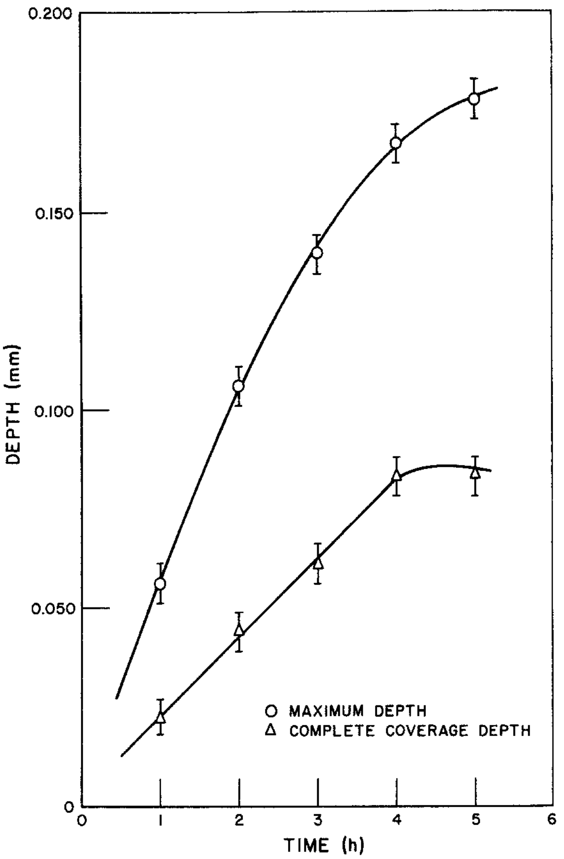

Figure 2 Boride layer depths as a function of time for AISI 1008 steel samples heated at 960° C in the pack mixture.

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Received 26 November
and accepted 12 December 1980

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The size of shear lips in polymers

Shear lips have been observed on the fracture surfaces of a number of polymeric materials, in particular polycarbonate [1–4], vinyl-urethane resins [5], polyphenylene oxide [6] and polyethylenes [7]. In other, more brittle materials such as polystyrene and polymethylmethacrylate (PMMA) shear lips are not observed. The normal equation used to estimate the width of a shear lip on a fracture surface, d, is that suggested by Irwin *et al.* [8, 9] for metallic materials

$$d = \frac{1}{2\pi}\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{y}}}\right)^2, \tag{1}$$

where σ_{y} is the tensile yield stress and K_{Ic} is the plane strain fracture toughness. This equation cannot describe or explain the situation where shear lips are not observed. Also there is no direct evidence that it is correct, even in form, in the situations where the shear lips are visible in polymers. The aim of this note is to follow up an earlier suggestion that shear lips form in crazing materials only when the stress across a craze is

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greater than the tensile yield stress of the material [5]. Kambour *et al.* [2] have expressed similar ideas when suggesting that shear lip width is controlled by the balance between the octahedral shear stress and the normal stress.

The Dugdale plastic zone [10] has been shown to be an accurate fit to the shape of the crack tip crazes in some polymer systems [11–13] and so it seems reasonable to describe the stresses around a crack tip craze by this model.

In deriving Equation 1 Irwin assumed that the width of shear lips on the specimen surface (r_p) was given by the distance from an elastic crack at which the stress reached the tensile yield stress. It was then assumed that the boundary between shear lip and undeformed material would intersect the fracture surface and specimen surface at 45° equating r_p and d. This latter assumption has been shown to be fairly accurate for two polymeric materials [2, 5] and Equation 1 has been shown to agree with experiment with a factor of two in some metals [14, 15].

It seems reasonable therefore to calculate the size of the shear lips in a material whose plane strain failure mode is crazing in a similar way but using the elastic stress field round a Dugdale craze zone rather than that round a sharp crack tip. It is assumed that r_p and hence d are given by the maximum distance from the crack plane at which the elastic stresses fulfill a plane stress yielding criterion. This is a little different from the technique used by Irwin as he calculated the distance in front of the crack tip (along the crack axis) at which the stress equalled the yield stress. In his case though the zone around the crack tip in which the stress was equal or greater than the yield stress was approximately circular so either technique would have given similar results. These elastic stresses were derived by Rice [16] and, using his notation, are given by

$$\sigma_{11} + \sigma_{22} = 4\,\mathrm{Re}\,[\phi'(z)]; \tag{2}$$

$$\sigma_{22} - i\sigma_{12} = 2\,\mathrm{Re}\,[\phi'(z)] + (z - \bar{z})\overline{\phi''(z)}; \tag{3}$$

where Re is the real part of the function and

$$\phi'(z) = \frac{\sigma_0}{\pi}\tan^{-1}\left|\left(\frac{R}{R-z}\right)^{1/2}\right|, \tag{4}$$

where the crack is on the half plane $x_2 = 0, x_1 \leqslant 0$ and the craze is at $x_2 = 0, 0 < x_1 \leqslant R$ so the craze length is R. The stress across the craze is σ_0 and $z = x_1 + ix_2$.

From Equation 4

$$\phi''(z) = \frac{\sigma_0}{\pi}\frac{i}{2z}\left(\frac{R}{R-z}\right)^{1/2}. \tag{5}$$

The von Mises yield criterion for the plane stress situation at the surface of the specimen is given by

$$\sigma_e \geqslant \sigma_y,$$

where

$$2\sigma_e^2 = (\sigma_{11} - \sigma_{22})^2 + \sigma_{11}^2 + \sigma_{22}^2 + 6\sigma_{12}^2 \tag{6}$$

and σ_y is the tensile yield stress of the material.

Substituting Equations 2 and 3 into Equation 6

$$\sigma_y^2 = 4(Re[\phi'(z)])^2 + 12x_2^2\,\phi''(z)\,\overline{\phi''(z)}. \tag{7}$$

Values of (σ_e/σ_0) were calculated for the region around a Dugdale zone and the results are shown in Fig. 1. The maximum value of σ_e/σ_0 for any given distance from the crack plane, x_2/R, occurs approximately at $x_1/R = 0.6$. We are assuming that r_y is given by the largest distance from the crack plane that the yield criterion is satisfied, so r_y equals the value of x_2 at $x_1/R \simeq 0.6$ for which $\sigma_e = \sigma_y$. This value of $x_2 = r_y$ is given in non-dimensional form as x_2/R in Fig. 2 as a function of σ_y/σ_0. We are equating the shear lip size (d) with r_y so Figs 1 and 2 show that the size of the shear lips is zero for the yield stress greater than the craze stress and increases steadily for decreasing yield stress less than the craze stress. Shear lips attain a size equal to the craze length, R, when $\sigma_y = 0.66\sigma_0$. It can be shown from the results in Fig. 2 that for values of $\sigma_y < 0.66\sigma_0$, r_y and d become independent of σ_0 and are given approximately by $0.2(K_I/\sigma_y)$ which is not significantly different from Equation 1.

Knott [17] has suggested that when allowance is made for the stress redistribution round a plastic zone, the plane stress plastic zone radius, r_y, should be given in non-crazing materials approximately by $0.4(K_I/\sigma_y)^2$ rather than Equation 1. There is some experimental evidence which supports this point of view [14]. If this is the case then one would expect the same effect to occur in crazing materials and so the shear lip sizes predicted in Figs 1 and 2 should be doubled.

The fracture energy of shear lips is considerably greater than that of the plane strain region in the

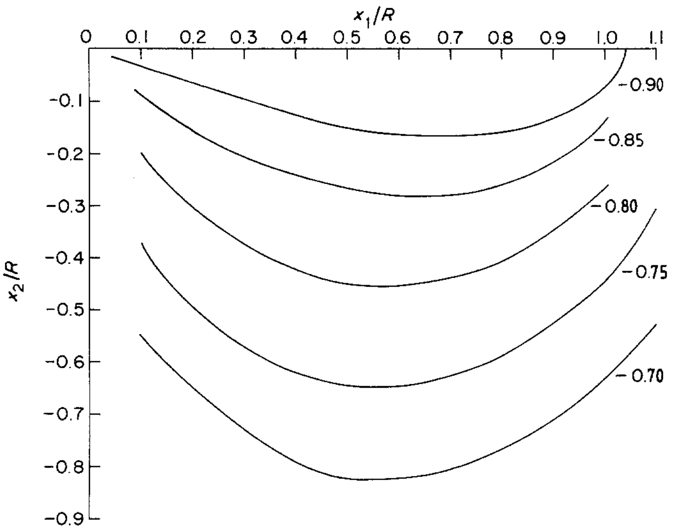

Figure 1 Contours of (σ_e/σ_0) around a crack tip craze.

centre of the specimen so even relatively small shear lips can cause the stress intensity required for crack propagation to be considerably greater than plane strain value. In this case the value of K_I used in Equation 1 has been the applied K_I [14, 15]. It is likely that in the same situation in polymers the applied K_I also should be used.

There is very little data with which the model described in this work can be compared. The basic assumption, that shear lips are only observed when the craze stress is greater than the yield stress, is followed in a number of systems. Shear lips are not observed in PMMA and polystyrene and in both systems crazes are clearly visible in tensile tests before yield occurs. In polycarbonate, however, crazes are not observed before yield in a tensile test, in polyethylene it is hard to know if they occur as the material is not clear. Shear lips are seen in both these materials. Shear lips do not occur, or are very small in polymers during high speed, unstable crack growth [2, 5] and so can only be expected to be significant in those systems in which the crack growth is stable. It is perhaps for this reason that they have not been reported in PVC in spite of the fact that some grades can yield in a tensile test without crazing.

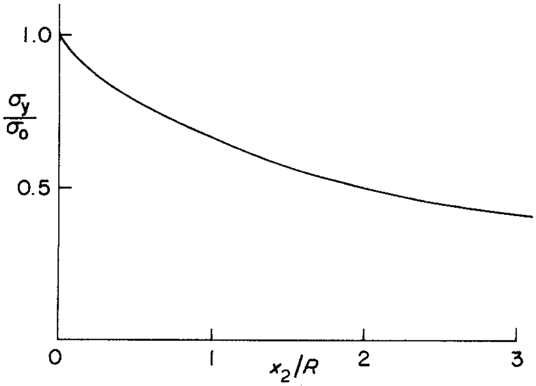

Figure 2 Non-dimensional plastic zone size x_2/R as a function of (σ_y/σ_0).

Shear lips have been reported in vinyl-urethane resins but in the systems described $\sigma_0 \sim 1.5\sigma_y$ so it is not possible to make a numerical comparison between this theory and Equation 1.

Pitman and Ward [4] examined the fracture of polycarbonate as a function of molecular weight at − 30° C. They found that shear lip size, craze length and craze stress (calculated using the Dugdale model) all decreased in size as the molecular weight was decreased by irradiation. The yield stress remained constant. The shear lip size decreased more rapidly than the craze length, however, the former changed by a factor of 10 whilst the latter changed by about a factor of 3. Qualitatively this agrees with the predictions of Fig. 2. The craze stress calculated by Pitman and Ward varies from greater than the yield stress at high molecular weights to less than the yield stress at low molecular weights; therefore, according to the model described here, shear lips should not have existed for the low molecular weight materials. The reason for this disagreement might be that the Dugdale model did not fit the craze shapes very well [5] so the craze stress probably varied considerably along the craze. Dugdale zone analysis gives some sort of average craze stress while the shear lip size may be given by the maximum craze stress if the latter occurs close to the craze tip. Parvin and Williams [1] used Equation 1 to estimate their shear lip sizes in polycarbonate but comment that "There is evidence of shear lips forming above -60° C and these reduce in size as the temperature falls. At all temperatures they are substantially smaller (< 0.1 mm) than the calculated r_{y2} (~ 1 mm)".

The stress across a crack tip craze, σ_0, need not be the same as the stress necessary to cause crazing in uniaxial tension. This is partly because the shapes of crazes without cracks and crack tip crazes are significantly different but there is also the difference in stress environment. The crack tip craze is in a plane strain situation where σ_{11} and σ_{22}, the stress along and normal to the craze, equal σ_0, and σ_{33} equals $2\nu\sigma_{11}$, where ν is Poisson's ratio. The effect of stress environment on crazing has been described by Sternstein and Ongchin [18] and also Oxborough and Bowden [19]. The forms of their crazing criteria are very similar and both fit the same biaxial data. The equation of Oxborough and Bowden can be evaluated in the triaxial situation for both the parameters which fit their polystyrene data and those from the PMMA data of Sternstein and Ongchin. One finds that the ratio of σ_0 to the uniaxial crazing stress, σ_c, equals 1.57 for polystyrene at room temperature and 1.08 for PMMA at 50° C. This result should be considered as evidence that σ_0 does not necessarily equal the uniaxial crazing stress σ_c.

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Received 21 August 1980
and accepted 7 January 1981

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